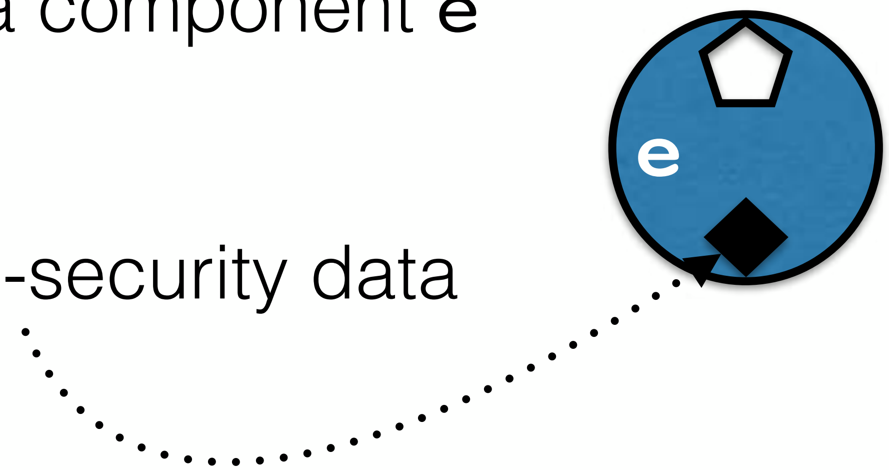
Noninterference for Free

William J. Bowman and Amal Ahmed

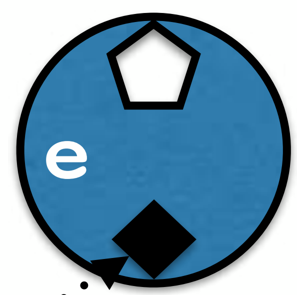
Let's write a secure program

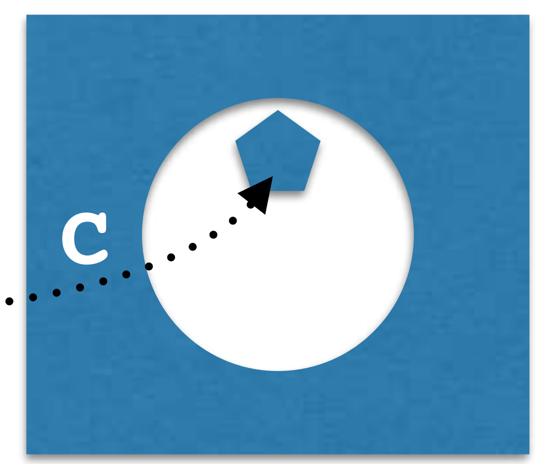
- Want to write a component **e** (browser)
- Manages high-security data (passwords)



Let’s write a secure program

- Want to write a component **e** (browser)
- Manages high-security data (passwords)
- Links with untrusted context **c** (plugins)
- **c** provides low-security inputs, reads low-security outputs





Language-based security!

Using language-based security,
we statically rule out attacks

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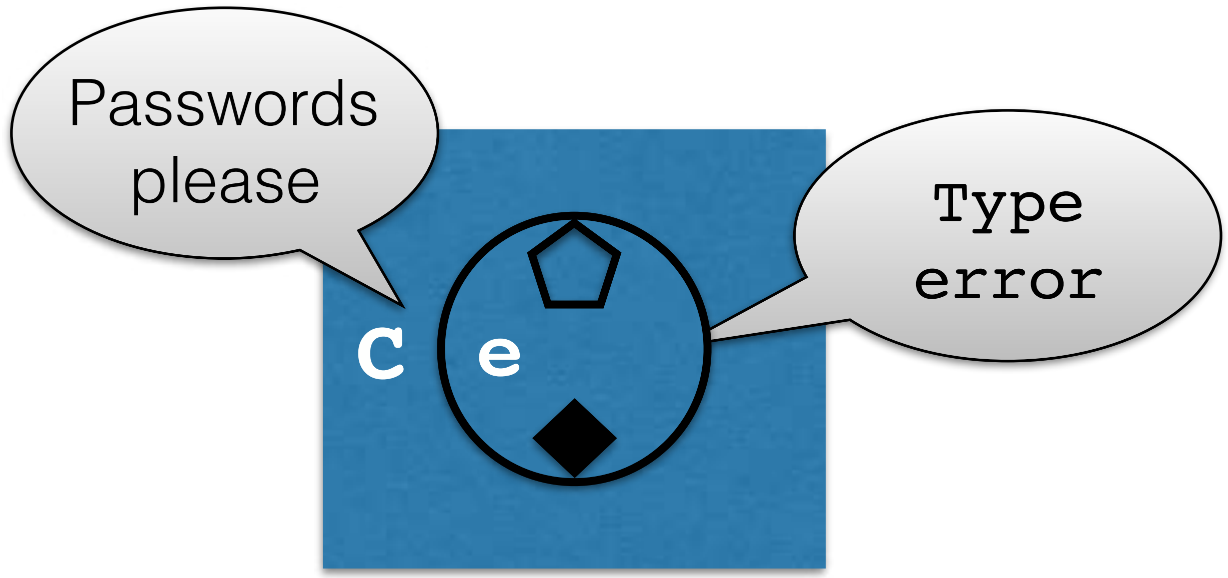
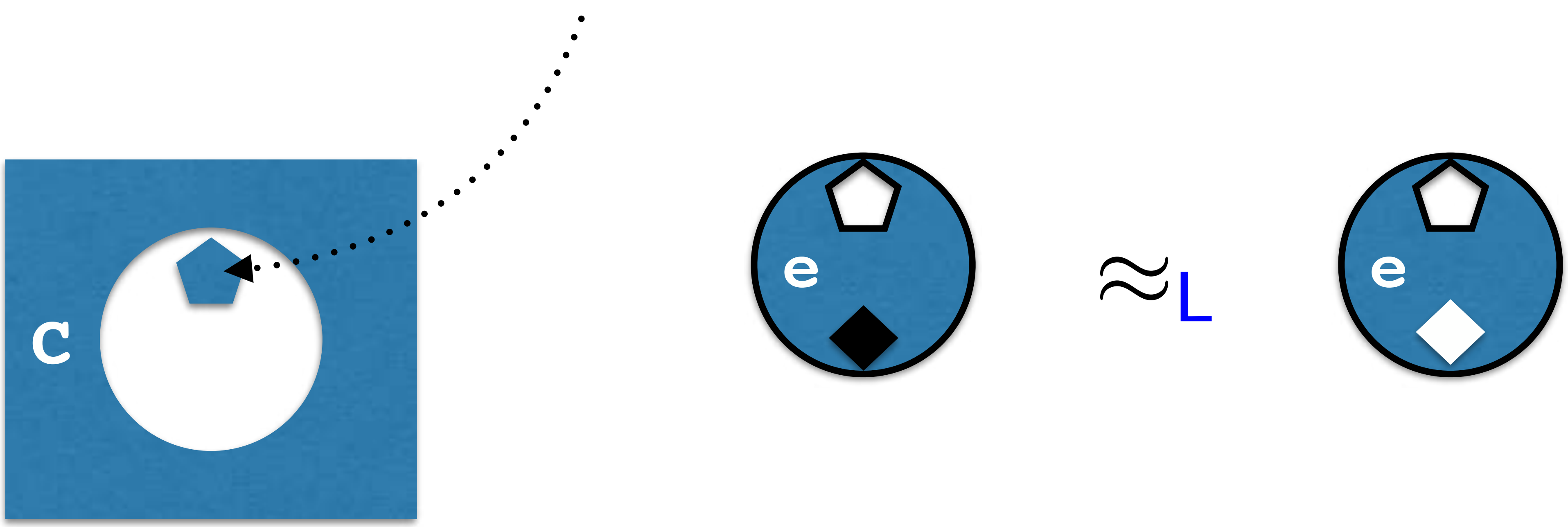


Using language-based security, we statically rule out attacks

Noninterference

Noninterference is an **equivalence property**
of any well-typed term **e**:

Given ***same*** low-level (***public***) inputs,



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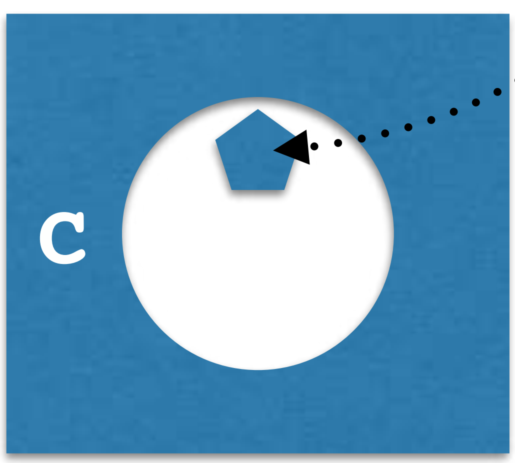
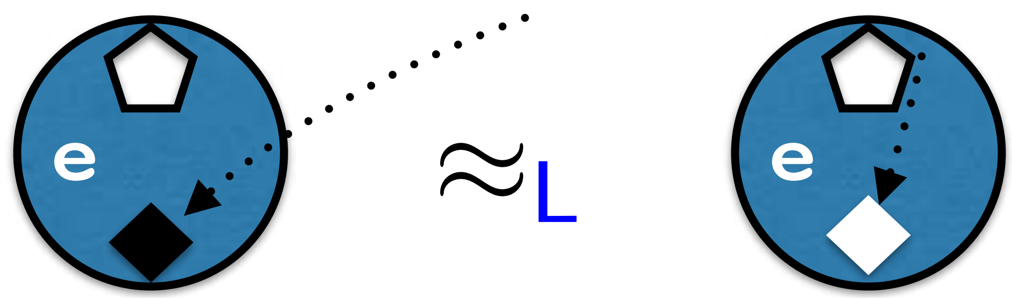
Given ***same*** low-level (***public***) inputs,

and ***different*** high-level (***private***) inputs





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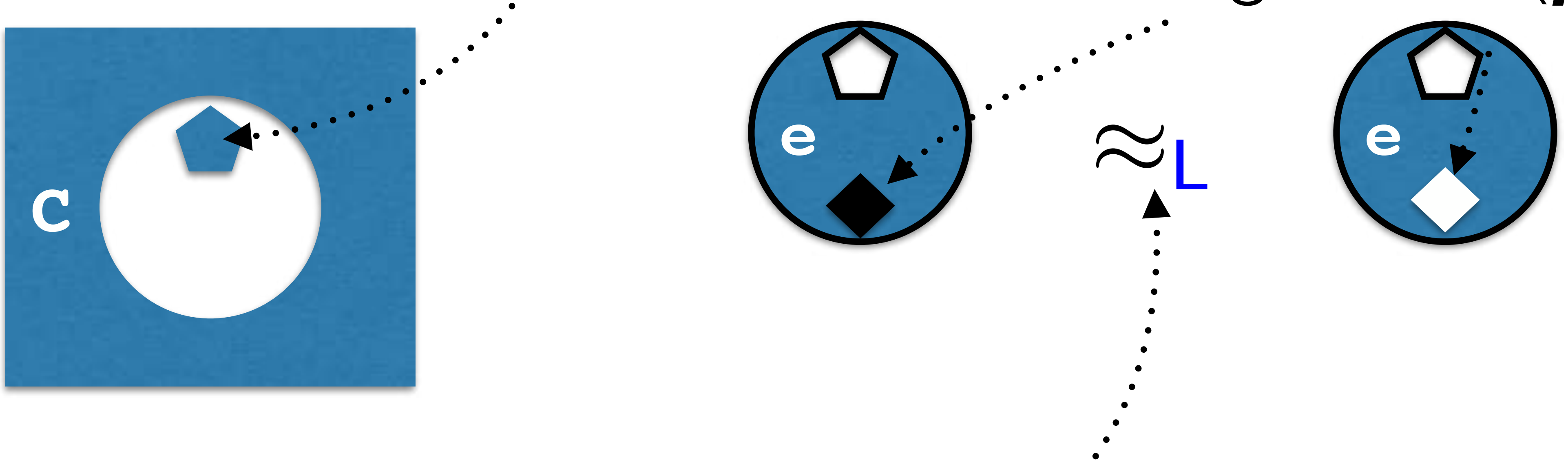

low-level outputs are indistinguishable

Noninterference

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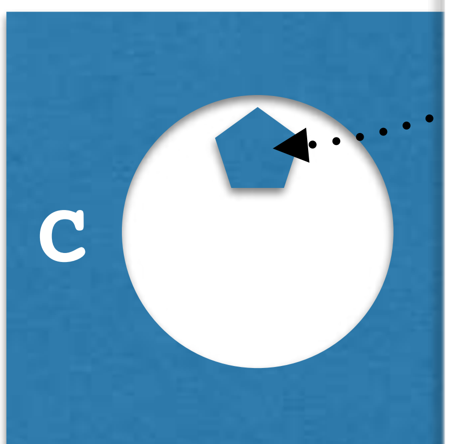
Given ***same***

vate) inputs

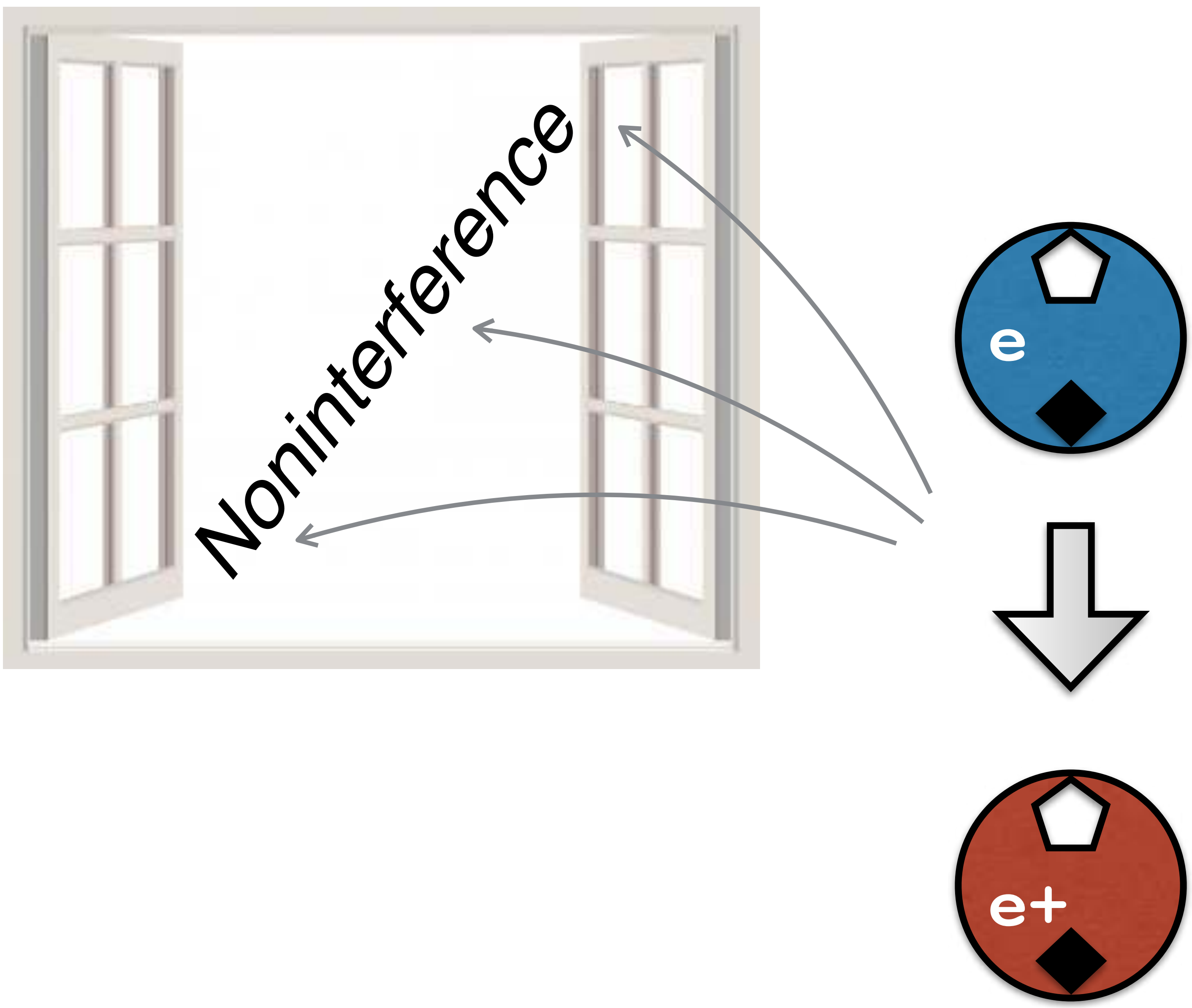


Security Solved!

low-level outputs are indistinguishable

WRONG

Because compilers



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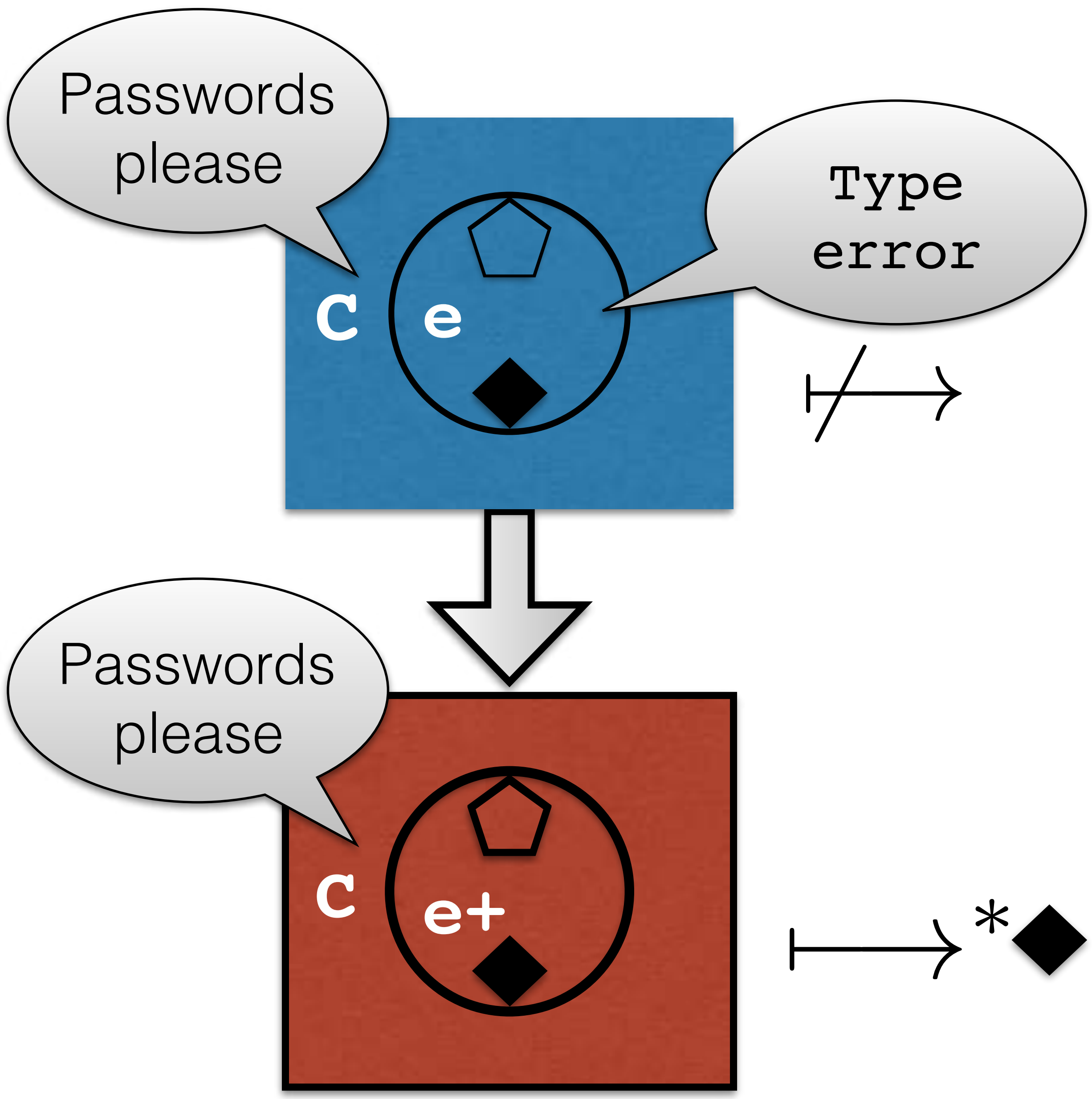
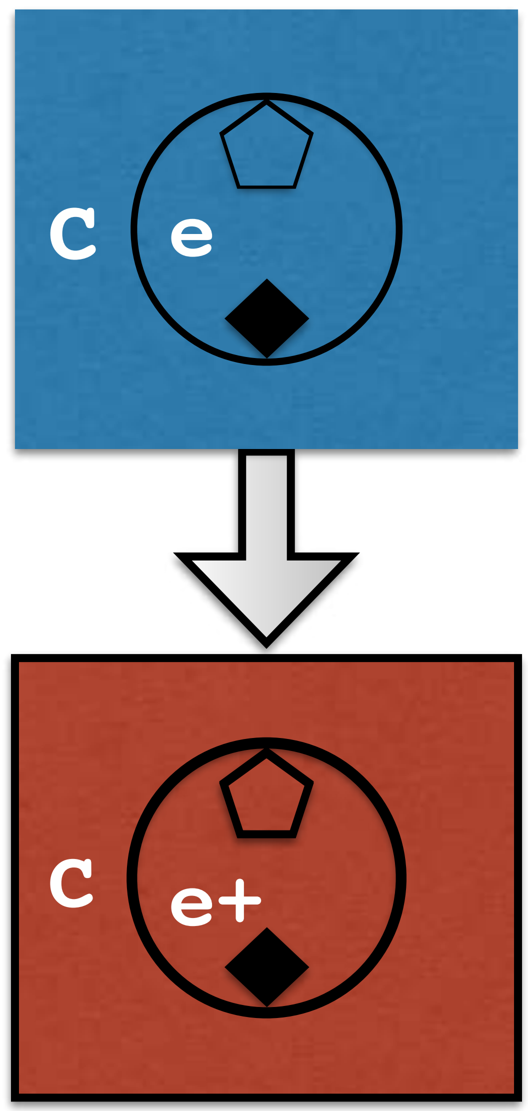


“Correct”

Even if the compiler is proven “correct”…

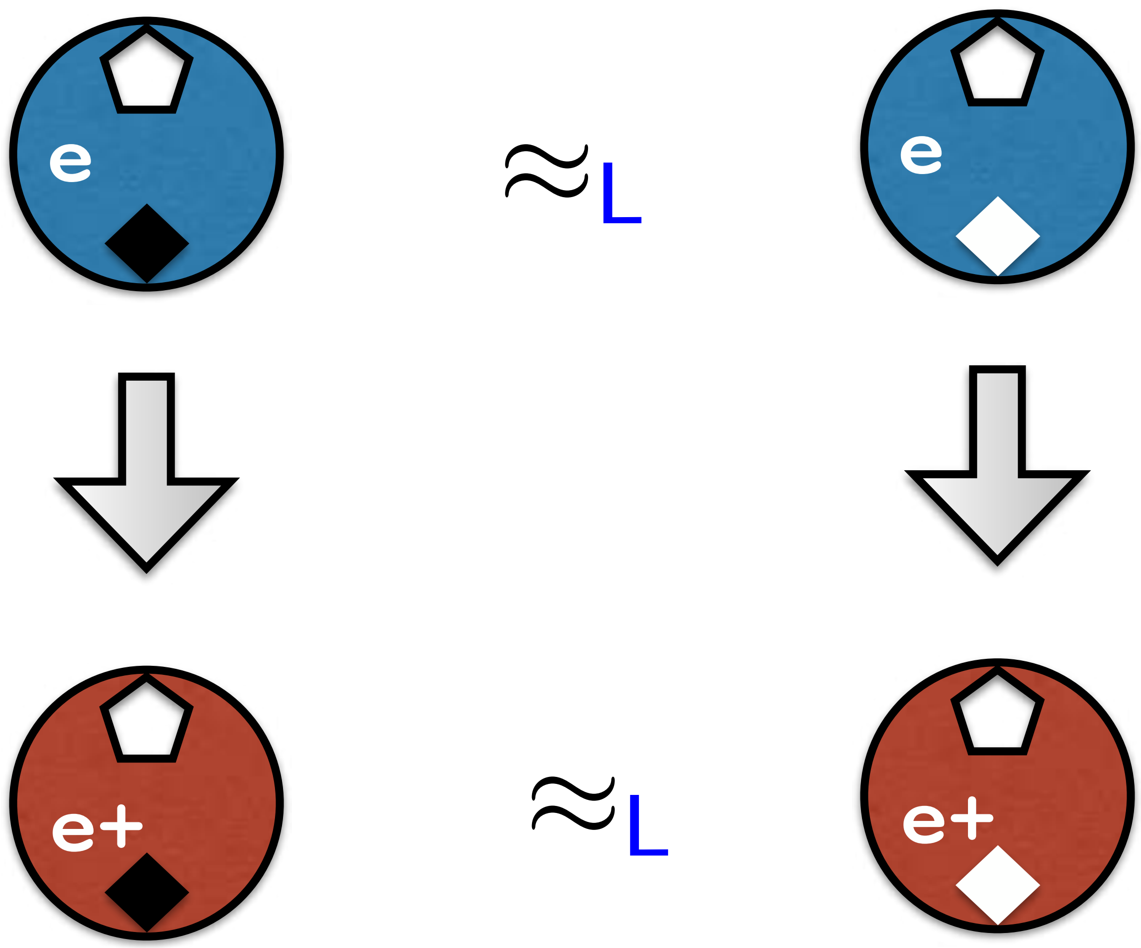


$$\longmapsto^* v$$

$$\longmapsto^* v^+$$

Equivalence Preserving

It may not preserve equivalences, e.g., noninterference.



How do we preserve noninterference?

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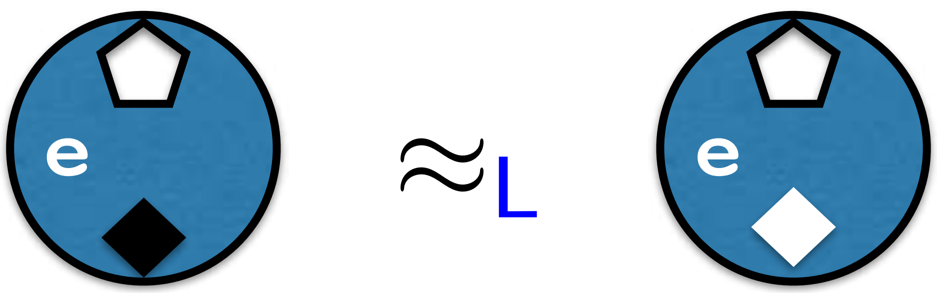
Folklore suggests noninterference can be



captured by parametricity

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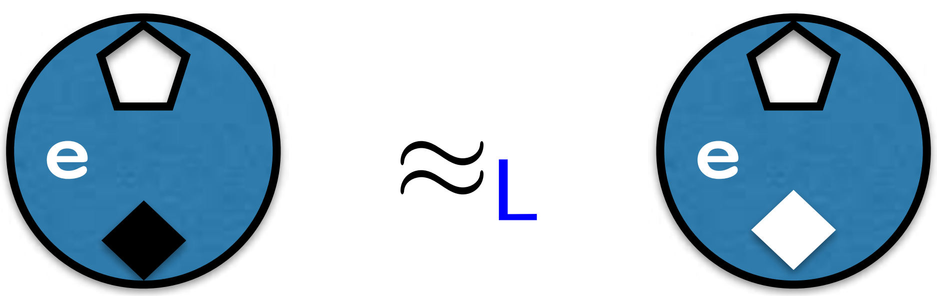

captured by parametricity

[1] Tse & Zdancewic, *Translating Dependency into Parametricity*, ICFP 2004

[2] Shikuma & Igarashi, *Proving Noninterference by a Fully Complete …*, ASIAN 2006

Languages

Source: DCC [1] → Target: System Fω

- Captures dependency analyses
  - e.g. Information-flow security
- STLC + Lattice of monads

- Parametricity
- Type constructors, i.e., higher-order polymorphism

[1] Abadi *et al.*, *The Core Calculus of Dependency,* POPL 1999

Source Language

DCC (Core Calculus of Dependency)

Monad protects
data based on label

$$\Gamma \vdash e_1 : T_\ell\, s_1$$

Source Language

DCC (Core Calculus of Dependency)

$\eta_{\mathsf{H}}\ \mathsf{true} \approx_{\mathsf{L}} \eta_{\mathsf{H}}\ \mathsf{false} : \mathsf{T_H\ bool}$

Monad protects
data based on label

$\Gamma \vdash \mathsf{e_1} : \mathsf{T}_\ell\ \mathsf{s_1}$

Source Language

DCC (Core Calculus of Dependency)

$$\frac{\Gamma \vdash e_1 : T_\ell\, s_1 \qquad \Gamma, x : s_1 \vdash e_2 : s_2}{\Gamma \vdash \mathsf{bind}\ x = e_1\ \mathsf{in}\ e_2 : s_2}$$

Term containing private data → e_1

Continuation using private data → e_2

Source Language

DCC (Core Calculus of Dependency)

Promise that result is protected

$$\frac{\Gamma \vdash e_1 : T_\ell\, s_1 \qquad \Gamma, x : s_1 \vdash e_2 : s_2 \qquad \ell \preceq s_2}{\Gamma \vdash \mathsf{bind}\, x = e_1\, \mathsf{in}\, e_2 : s_2}$$

Source Language

DCC (Core Calculus of Dependency)

Promise that result is protected

For example…

$\mathsf{L} \not\preceq \mathsf{bool}$ $\quad$ $\mathsf{H} \preceq 1$

$\ell \preceq \mathsf{s}_2$

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DCC (Core Calculus of Dependency)

Promise that result is protected

$\ell \preceq s_2$

For example…

$L \not\preceq \text{bool}$ $\quad$ $H \preceq 1$

$L \preceq T_L\ s$ $\quad$ $H \not\preceq T_L\ s$

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Term containing private data

Continuation using private data

Translation, Briefly

How are types translated?

$$1^+ = \mathbf{1}$$

$$\mathsf{bool}^+ = \mathbf{bool}$$

$$(\mathsf{s}_1 \rightarrow \mathsf{s}_2)^+ = \mathsf{s}_1^+ \rightarrow \mathsf{s}_2^+$$

$$(\mathsf{T}_\ell\ \mathsf{s})^+ = \ ?$$

Translation, Briefly

$$\frac{\Gamma \vdash \mathsf{e}_1 : \mathsf{T}_\ell\ \mathsf{s}_1 \qquad \Gamma, \mathsf{x} : \mathsf{s}_1 \vdash \mathsf{e}_2 : \mathsf{s}_2 \qquad \ell \preceq \mathsf{s}_2}{\Gamma \vdash \mathsf{bind}\ \mathsf{x} = \mathsf{e}_1\ \mathsf{in}\ \mathsf{e}_2 : \mathsf{s}_2}$$

Idea: CPS the monad + constrain continuation result

$$(\mathsf{T}_\ell\ \mathsf{s})^+ = \forall \boldsymbol{\beta} :: *. (\mathsf{s}^+ \to \boldsymbol{\beta}) \to \boldsymbol{\beta}$$
$$\text{s.t. } \ell \preceq \boldsymbol{\beta}$$

Translation, Briefly

$$\frac{\Gamma \vdash e_1 : T_\ell\, s_1 \qquad \Gamma, x : s_1 \vdash e_2 : s_2 \qquad \ell \preceq s_2}{\Gamma \vdash \text{bind } x = e_1 \text{ in } e_2 : s_2}$$

$$(T_\ell\, s)^+ = \forall \boldsymbol{\beta}{::}{*}.\, (\llbracket \ell \preceq \boldsymbol{\beta} \rrbracket \times (s^+ \rightarrow \boldsymbol{\beta})) \rightarrow \boldsymbol{\beta}$$

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$$[\![\ell \preceq s^+]\!] = (\alpha_\preceq\ \ \alpha_\ell\ \ s^+)$$

Translation, Briefly

$$\frac{\Gamma \vdash e_1 : T_\ell\, s_1 \qquad \Gamma, x : s_1 \vdash e_2 : s_2 \qquad \ell \preceq s_2}{\Gamma \vdash \mathsf{bind}\; x = e_1 \;\mathsf{in}\; e_2 : s_2}$$

$$(T_\ell\, s)^+ = \forall \beta{::}{*}.\,((\alpha_{\preceq}\;\; \alpha_\ell\;\; \beta) \times (s \to \beta)) \to \beta$$

$$
\begin{array}{l}
\mathbf{data}\;\; (\alpha_{\preceq}\;\; \alpha_\ell\;\; s^+)\;\; \mathbf{where} \\
\quad \mathtt{Unit\_P} :: (\alpha_{\preceq}\;\; \alpha_\ell\;\; 1) \\
\quad \ldots \\
\quad \mathtt{Monad\_P} :: [\![\ell \sqsubseteq \ell']\!] \to (\alpha_{\preceq}\;\; \alpha_\ell\;\; (T_{\ell'}\, s)^+)
\end{array}
$$

Translation Summary

$$1^+ = \mathbf{1}$$

$$\mathsf{bool}^+ = \mathbf{bool}$$

$$(\mathsf{s}_1 \to \mathsf{s}_2)^+ = \mathsf{s}_1^+ \to \mathsf{s}_2^+$$

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Invariants:

$\vdash \mathsf{s}$ $\qquad\qquad$ $\boldsymbol{\alpha}_{\preceq}, \boldsymbol{\alpha}_\ell, \cdots \vdash \mathsf{s}^+$

Proving Noninterference Preservation

Proving Equivalence Preservation

Equiv. Preservation is Hard

To *show*

$$\lambda x : s.\, e \approx \lambda x : s.\, e'$$

implies

$$\boldsymbol{\lambda x}{:}s^{+}.e^{+} \approx \boldsymbol{\lambda x}{:}s^{+}.e'^{+}$$

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Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$

Show $e^{+}[\mathbf{m_1}/\mathbf{x}] \approx e'^{+}[\mathbf{m_2}/\mathbf{x}]$

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- By assumption, $\lambda x : s.\, e \approx \lambda x : s.\, e'$
- Hence, $e[\mathbf{m_1}/x] \approx e'[\mathbf{m_2}/x]$
- By induction

Equivalence Reflection?

How do we say that since $\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^+$

$$\begin{array}{ccc} \mathbf{m_1} & & \mathbf{m_2} \\ \wr & & \wr \\ \mathsf{e}_1 & & \mathsf{e}_2 : \mathsf{s} \end{array}$$

there must exist $\mathsf{e}_1 \quad \mathsf{e}_2 : \mathsf{s}$

Equivalence Reflection?

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$$\wr \wr \qquad \wr \wr$$

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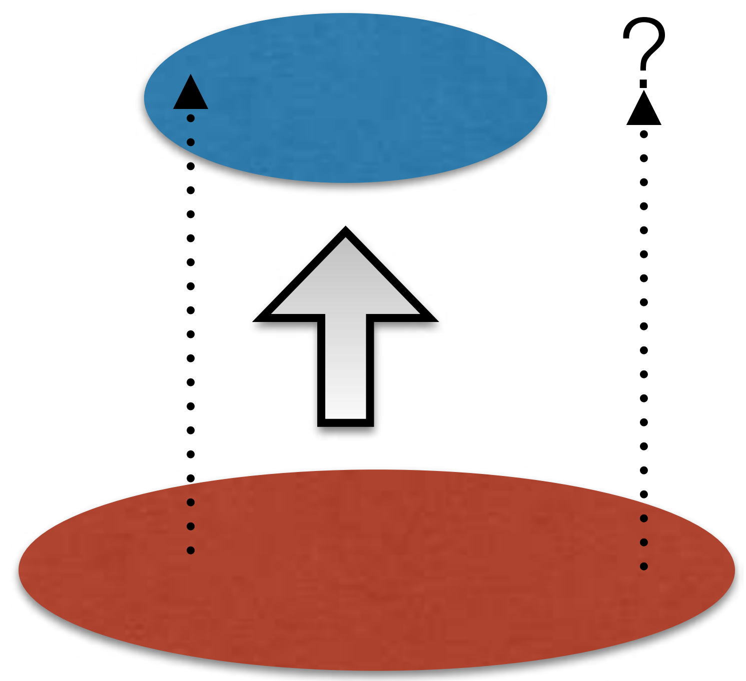
Back-translation

How can we possibly back-translate a *more expressive* target language?



Enrich Source?

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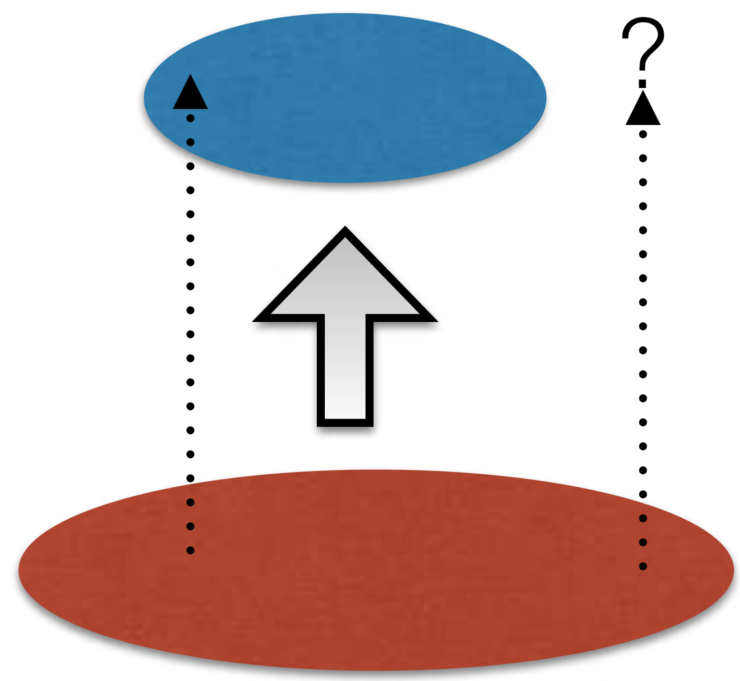

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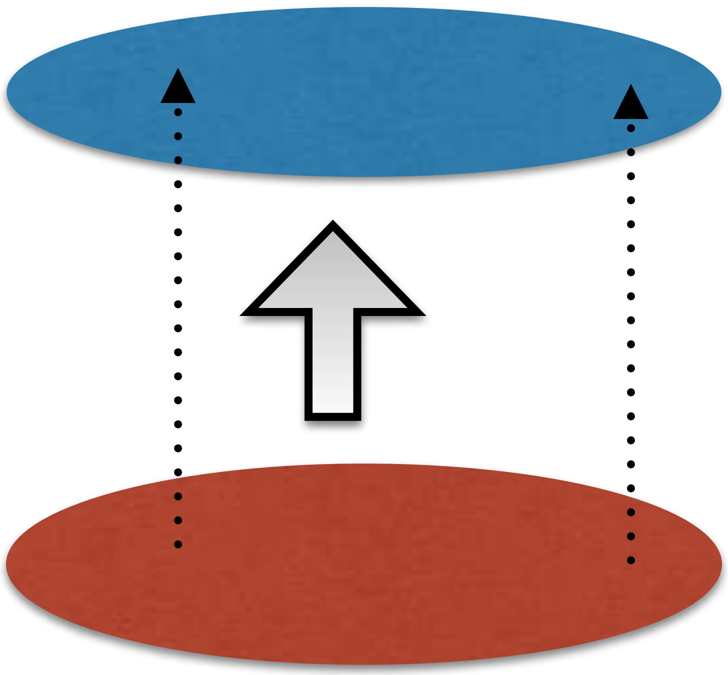

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Impoverish Target?

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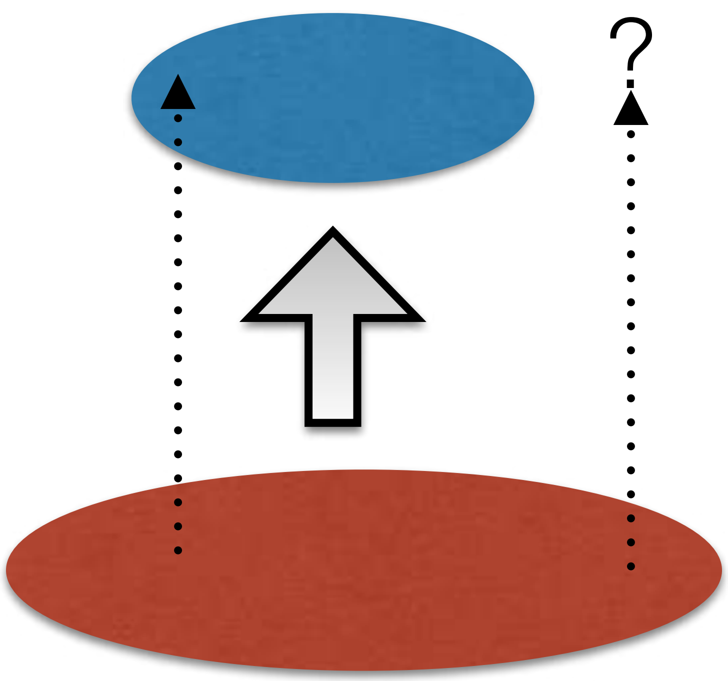

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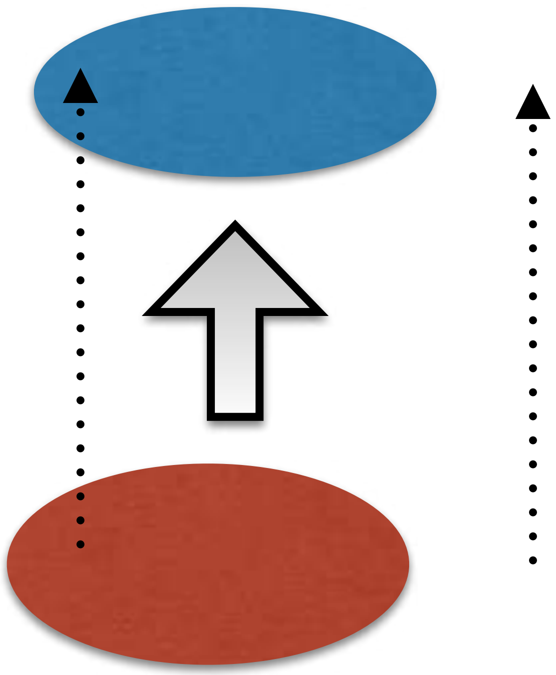
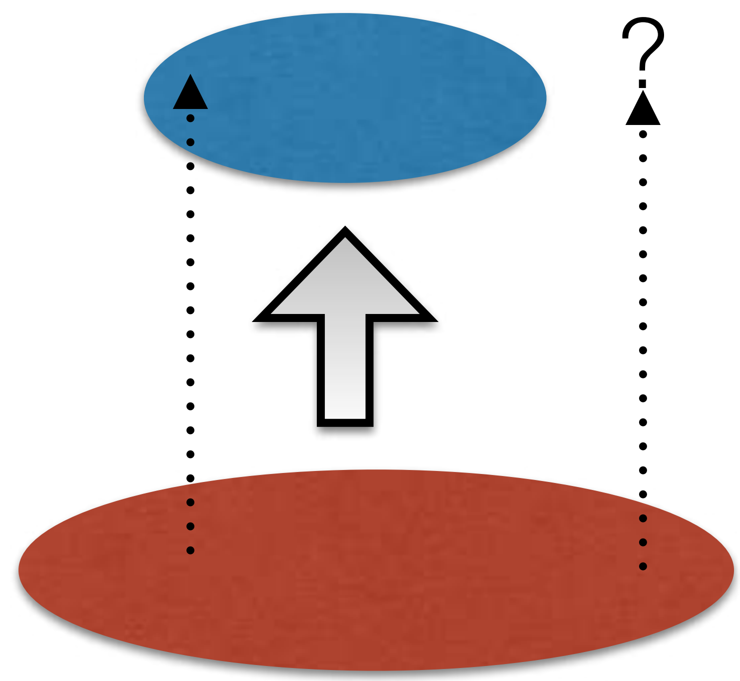


Or impoverish the target

None of the above

How can we possibly back-translate
a *more expressive* target language?

Neither is satisfying



Be clever

How can we possibly back-translate
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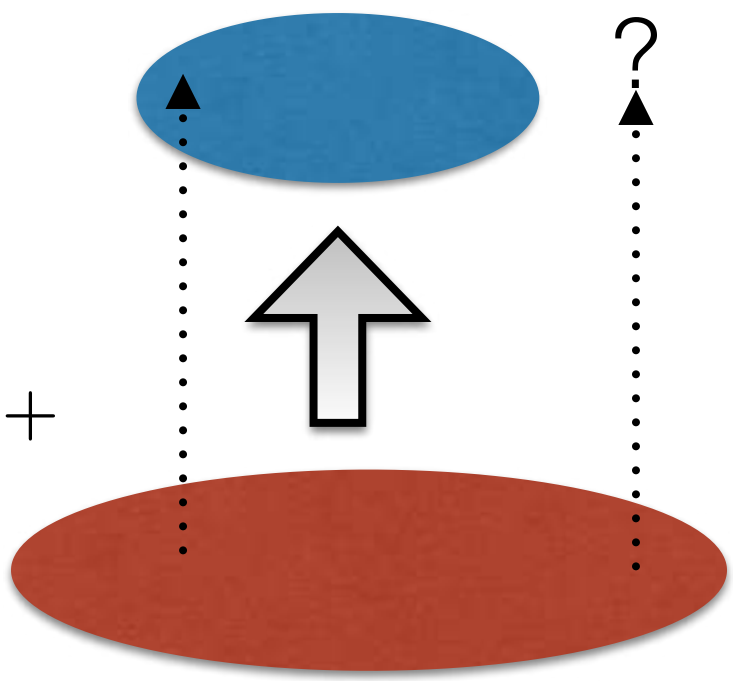
Recall:
Assume $\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^+$



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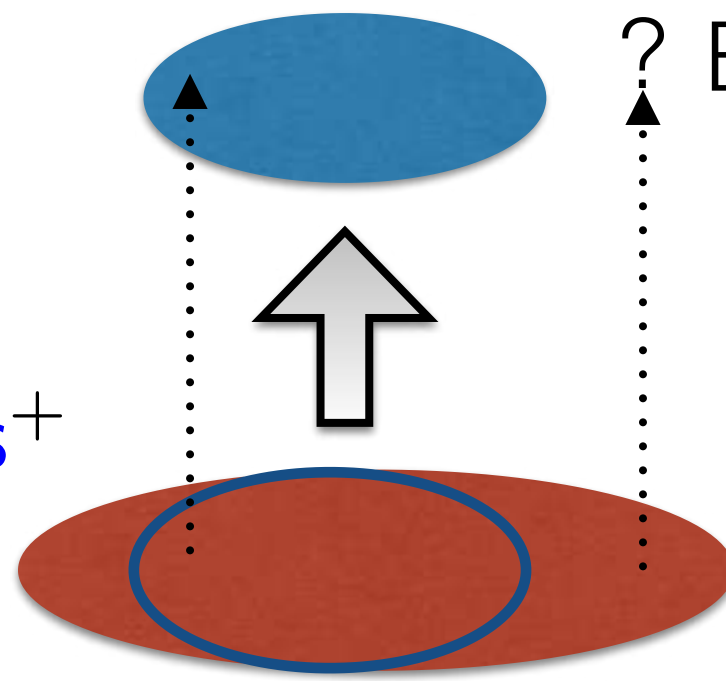

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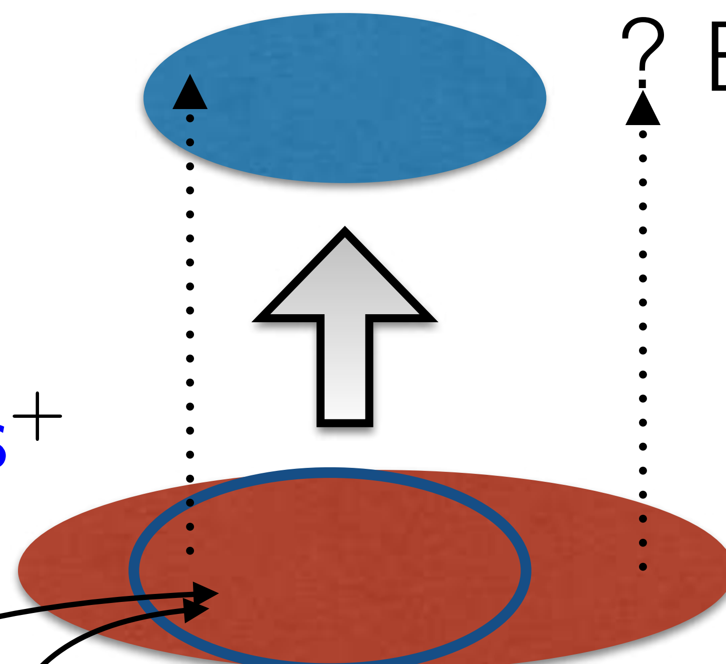

? By using types.

Recall:
Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$

e.g.

$$\mathbf{x} : s_1^{+} \vdash \mathbf{m} : s_2^{+} \quad \cdots\cdots\blacktriangleright \quad x : s_1 \vdash e : s_2$$

$$\boldsymbol{\lambda}\mathbf{x}{:}s_1^{+}.\mathbf{m} : s_1^{+} \rightarrow s_2^{+} \quad \cdots\cdots\blacktriangleright \quad \lambda x : s_1 . e : s_1 \rightarrow s_2$$

Be clever

How can we possibly back-translate
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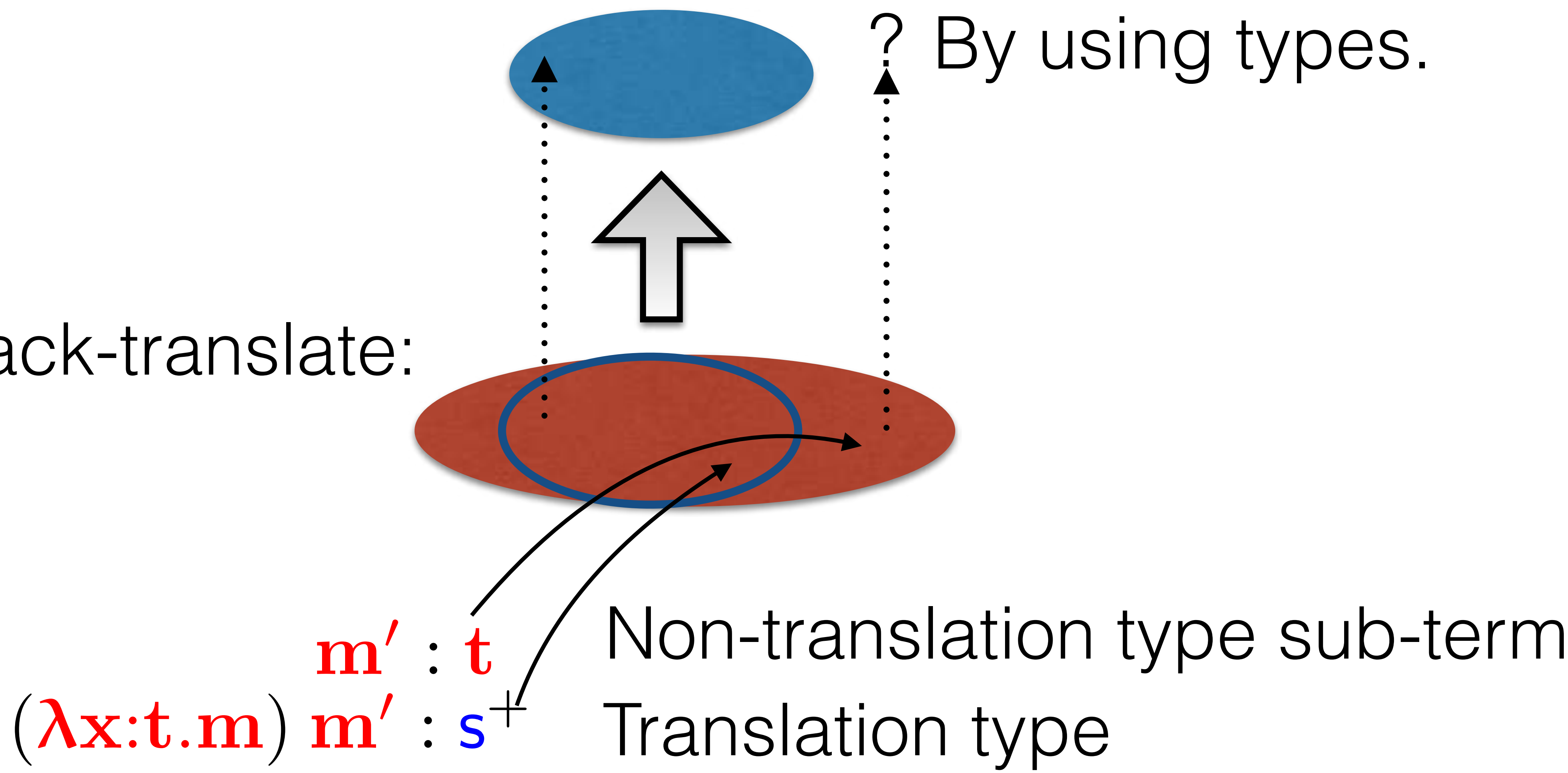
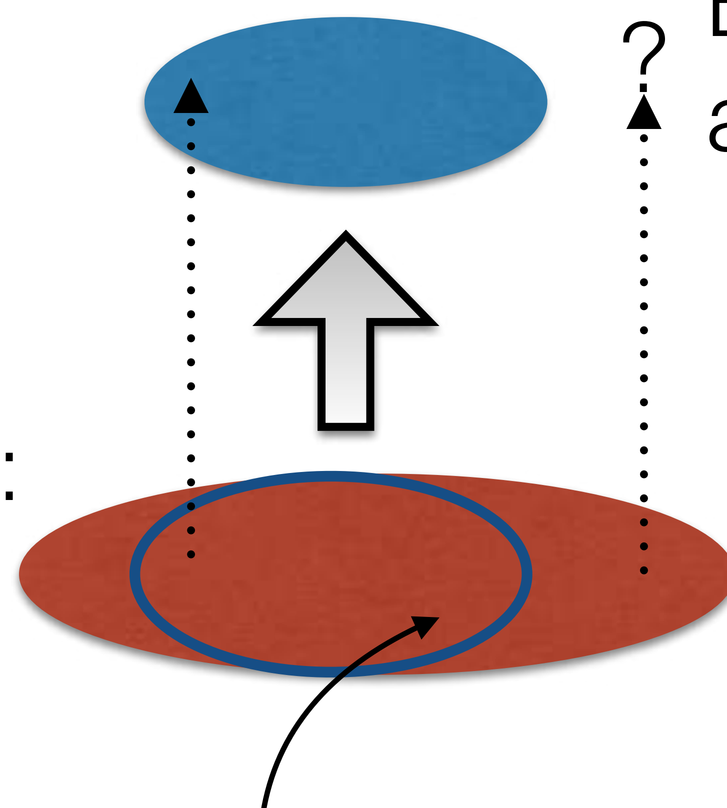
Can we back-translate:



Be cleverer

How can we possibly back-translate
a *more expressive* target language?

By using types
and partial evaluation.



Can we back-translate:

Not by induction

$(\lambda x{:}t.m)\ m' \longmapsto m'' : s^+$

$m' : t$ is no more

Be clevererer

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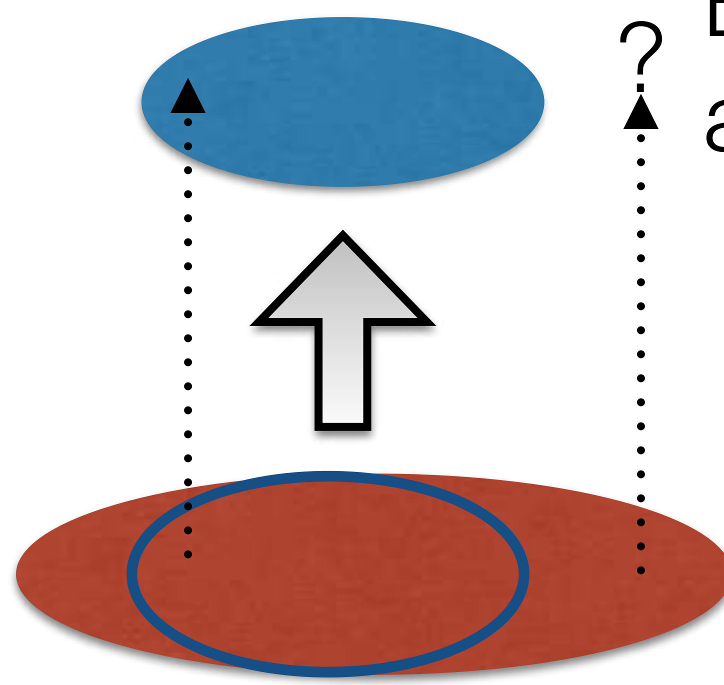

Not by induction

Therefore, must prove all terms are back-translatable.

Proof of Equiv. Preservation

To *show*

$$\lambda x : s.\, e \approx \lambda x : s.\, e'$$

implies

$$\boldsymbol{\lambda x{:}}s^+\boldsymbol{.}e^+ \approx \boldsymbol{\lambda x{:}}s^+\boldsymbol{.}e'^+$$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^+$

Show $e^+[\mathbf{m_1}/\mathbf{x}] \approx e'^+[\mathbf{m_2}/\mathbf{x}]$

How the proof proceeds:

Proof of Equiv. Preservation

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How the proof proceeds:

- Back-translate $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$ to $e_1 \approx e_2 : s$

Proof of Equiv. Preservation

To *show*

$$\lambda x:s.\,e \approx \lambda x:s.\,e'$$

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How the proof proceeds:

- Back-translate $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$ to $e_1 \approx e_2 : s$
- By assumption, $\lambda x:s.\,e \approx \lambda x:s.\,e'$

Proof of Equiv. Preservation

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Proof of Equiv. Preservation

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- Back-translate $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$ to $e_1 \approx e_2 : s$
- By assumption, $\lambda x : s.\, e \approx \lambda x : s.\, e'$
- Hence, $e[e_1/x] \approx e'[e_2/x]$
- By induction. QED.

Proof of Equiv. Preservation

To *show*

$\lambda x : s.\, e \approx \lambda x : s.\, e'$

implies

$\lambda x{:}s^+.e^+ \approx \lambda x$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^+$

Show $e^+[\mathbf{m_1}/\mathbf{x}] \approx e'^+[\mathbf{m_2}/\mathbf{x}]$

roceeds:

$_1 \approx \mathbf{m_2} : s^+$

$: s.\, e \approx \lambda x : s.\, e'$

$e'[e_2/x]$

.

EXCEPT

Proof of Equiv. Preservation

To *show*

$\lambda x : s.\, e \approx \lambda x : s.\, e'$

implies

$\lambda \mathbf{x} : s^{+} .\, e^{+} \approx \lambda \mathbf{x}$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$

Show $e^{+}[\mathbf{m_1}/\mathbf{x}] \approx e'^{+}[\mathbf{m_2}/\mathbf{x}]$

oes:

$\mathbf{{}_1} \approx \mathbf{m_2} : s^{+}$

$x : s.\, e \approx \lambda x : s.\, e'$

$e'[e_2/x]$

.

EXCEPT

If you don't understand logical relations, you can stop listening for the next 4 slides.

“Open” Logical Relations

Typically, logical relations are defined on closed terms/types.

Again recall: Assume $\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^+$

And: $\boldsymbol{\alpha}_{\preceq}, \boldsymbol{\alpha}_{\ell}, \cdots \vdash \mathsf{s}^+$

But translation types are only well-formed when open.

“Open” Logical Relations

Again recall: Assume $\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^+$

And: $\boldsymbol{\alpha}_{\preceq}, \boldsymbol{\alpha}_{\ell}, \cdots \vdash \mathsf{s}^+$

i.e., to even *state* this assumption, the logical relation *must* leave these type variables open.

"Open" Logical Relations

~~$\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^{+}$~~

building on [1], we define $\mathbf{m_1} \approx^{\mathbf{\Sigma}} \mathbf{m_2} : \mathsf{s}^{+}$

$\mathbf{\Sigma} =$

```
data (α_⪯  α_ℓ  s⁺) where
  Unit_P :: (α_⪯  α_ℓ  1)
  ...
  Monad_P :: (α_⪯  α_ℓ  (T_ℓ s)⁺)
```

[1] Zhao *et al.*, *Relational Parametricity for Linear System F*, APLAS 2010

QED! (ish)

$$\sout{\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^+}$$

$$\mathbf{m_1} \approx^{\boldsymbol{\Sigma}} \mathbf{m_2} : \mathsf{s}^+$$

$$\boldsymbol{\Sigma} =$$

```
data (α_⪯ α_ℓ s⁺) where
  Unit_P :: (α_⪯ α_ℓ 1)
  ...
  Monad_P :: (α_⪯ α_ℓ (T_ℓ s)⁺)
```

$$[\![\boldsymbol{\Sigma}]\!]_{\mathsf{L}} = ?$$

Conclusion

- Language-based reasoning requires better compilers
- We have developed techniques for such compilers (specifically, for noninterference preservation)

https://www.williamjbowman.com/papers#niforfree